Time dilation

Time dilation explains why two working clocks will report different times after different accelerations. For example, ISS astronauts return from missions having aged slightly less than they would have been if they had remained on Earth, and GPS satellites work because they adjust for similar bending of spacetime to coordinate with systems on Earth.[1]

In the theory of relativity, **time dilation** is a difference of elapsed time between two events as measured by observers either moving relative to each other or differently situated from a gravitational mass or masses.

An accurate clock at rest with respect to one observer may be measured to tick at a different rate when compared to a second observer's own equally accurate clock. This effect arises neither from technical aspects of the clocks nor from the fact that signals need time to propagate, but from the nature of spacetime itself.

1 Overview

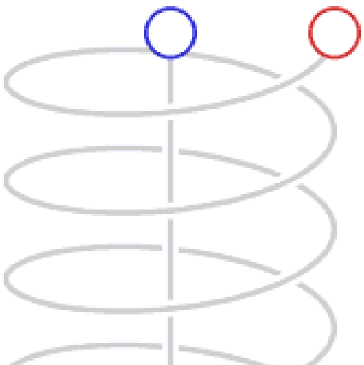
Clocks on the Space Shuttle ran slightly slower than reference clocks on Earth, while clocks on GPS and Galileo satellites run slightly faster.[1] Such time dilation has been repeatedly demonstrated (see experimental confirmation below), for instance by small disparities in atomic clocks on Earth and in space, even though both clocks work perfectly (it is not a mechanical malfunction). The laws of nature are such that time itself (i.e. spacetime) will bend due to differences in either gravity or velocity – each of which affects time in different ways.[2][3]

In theory, and to make a clearer example, time dilation could affect planned meetings for astronauts with advanced technologies and greater travel speeds. The astronauts would have to set their clocks to count exactly 80 years, whereas mission control – back on Earth – might need to count 81 years. The astronauts would return to Earth, after their mission, having aged one year less than the people staying on Earth. What is more, the local experience of time passing never actually changes for anyone. In other words, the astronauts on the ship as well as the mission control crew on Earth each feel normal, despite the effects of time dilation (i.e. to the traveling party, those stationary are living “faster"; while to those who stood still, their counterparts in motion live “slower” at any given moment).

With technology limiting the velocities of astronauts, these differences are minuscule: after 6 months on the International Space Station (ISS), the astronaut crew has indeed aged less than those on Earth, but only by about 0.005 seconds (nowhere near the 1 year disparity from the theoretical example). The effects would be greater if the astronauts were traveling nearer to the speed of light (299,792,458 m/s), instead of their actual speed – which is the speed of the orbiting ISS, about 7,700 m/s.[4]

Time dilation is caused by differences in either gravity or relative velocity. In the case of ISS, time is slower due to the velocity in circular orbit; this effect is slightly reduced by the opposing effect of less gravitational potential.

1.1 Relative velocity time dilation



From the local frame of reference (the blue clock), the relatively accelerated red clock moves slower

When two observers are in relative uniform motion and uninfluenced by any gravitational mass, the point of view of each will be that the other's (moving) clock is ticking at a *slower* rate than the local clock. The faster the relative velocity, the greater the magnitude of time dilation. This case is sometimes called special relativistic time dilation.

For instance, two rocket ships (A and B) speeding past one another in space would experience time dilation. If they somehow had a clear view into each other's ships, each crew would see the others' clocks and movement as going more slowly. That is, inside the frame of reference of Ship A, everything is moving normally, but everything over on Ship B appears to be moving more slowly (and vice versa).

From a local perspective, time registered by clocks that are at rest with respect to the local frame of reference (and far from any gravitational mass) always appears to pass at the same rate. In other words, if a new ship, Ship C, travels alongside Ship A, it is "at rest" relative to Ship A. From the point of view of Ship A, new Ship C's time would appear normal too.[5]

A question arises: If Ship A and Ship B both think each other's time is moving slower, who will have aged more if they decided to meet up? With a more sophisticated understanding of relative velocity time dilation, this seeming twin paradox turns out not to be a paradox at all (the resolution of the paradox involves a jump in time, as a result of the accelerated observer turning around). Similarly, understanding the twin paradox would help explain why astronauts on the ISS age slower (e.g. 0.007 seconds behind for every six months) even though they are experiencing relative velocity time dilation.

1.2 Gravitational time dilation

Main article: Gravitational time dilation

Gravitational time dilation is at play for ISS astronauts too, and it has the opposite effect of the relative velocity time dilation. To simplify, relative velocity and gravity each slow down time as they increase. Velocity has increased for the astronauts, slowing down their time, whereas gravity has decreased, speeding up time (the astronauts are experiencing less gravity than on Earth). Nevertheless, the ISS astronaut crew ultimately end up with "slower" time because the two opposing effects are not equally strong. The velocity time dilation (explained above) is making a bigger difference, and slowing down time. The (time-speeding up) effects of low-gravity would not cancel out these (time-slowing down) effects of velocity unless the ISS orbited much farther from Earth.

The key is that both observers are differently situated in their distance from a significant gravitational mass. The general theory of relativity describes how, for both observers, the clock that is closer to the gravitational mass, i.e. deeper in its "gravity well", appears to go more slowly than the clock that is more distant from the mass. This effect is not restricted to astronauts in space; a climber's time is passing slightly faster at the top of a mountain (a

Time passes more quickly further from a center of gravity, as is witnessed with massive objects (like the Earth).

high altitude, farther from the Earth's center of gravity) compared to people at sea level. As with all time dilation, the local experience of time is normal (nobody notices a difference within their own frame of reference). In the situations of velocity time dilation, both observers saw the other as moving slower (a reciprocal effect). Now, with gravitational time dilation, both observers – those at sea level, versus the climber – agree that the clock nearer the mass is slower in rate, and they agree on the ratio of the difference (time dilation from gravity is therefore not reciprocal). That is, the climber sees the sea level clocks as moving more slowly, and those living at sea level see the climber's clock as moving faster.

1.3 Time dilation: special vs. general theories of relativity

In Albert Einstein's theory of relativity, time dilation in these two circumstances can be summarized:

- In special relativity (or, hypothetically far from all gravitational mass), clocks that are moving with respect to an inertial system of observation are measured to be running more slowly. This effect is described precisely by the Lorentz transformation.
- In general relativity, clocks at a position with lower gravitational potential – such as in closer proximity to a planet – are found to be running more slowly. The articles on gravitational time dilation and gravitational redshift give a more detailed discussion.

Special and general relativistic effects can combine (as seen with ISS astronauts).

In special relativity, the time dilation effect is reciprocal: as observed from the point of view of either of two clocks which are in motion with respect to each other, it will be the other clock that is time dilated. (This presumes that the relative motion of both parties is uniform; that is, they do not accelerate with respect to one another during the course of the observations.) In contrast, gravitational time dilation (as treated in general relativity) is not reciprocal: an observer at the top of a tower will observe that clocks at ground level tick slower, and observers on the ground will agree about the direction and the magnitude of the difference. There is still some disagreement in a sense, because all the observers believe their own local clocks are correct, but the direction and ratio of gravitational time dilation is agreed by all observers, independent of their altitude.

1.4 Science fiction implications

Science fiction enthusiasts have noted the implications time dilation has on forward time travel, technically making it possible.[6]The Hafele and Keating experiment involved flying planes around the world with atomic clocks on board. Upon the trips' completion the clocks were compared to a static, ground based atomic clock. It was found that 273+/−7 nanoseconds had been gained on the planes' clocks.[7] The current human time travel record holder is Russian cosmonaut Sergei Krikalev,[8] who beat the previous record of about 20 milliseconds by cosmonaut Sergei Avdeyev.[9]

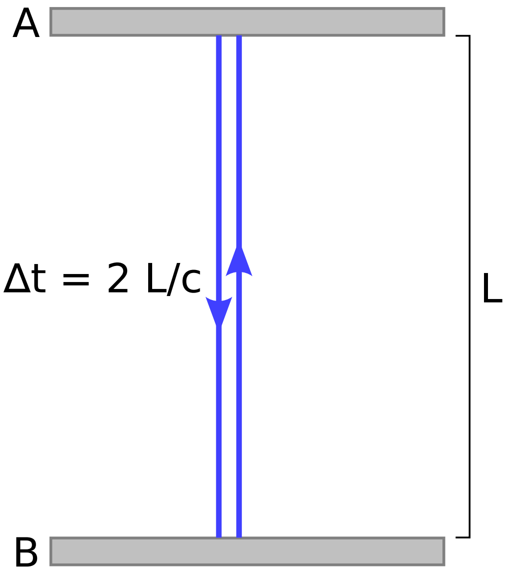
2 Simple inference of time dilation due to relative velocity

Time dilation can be inferred from the observed fact of the constancy of the speed of light in all reference frames.[10][11][12][13]

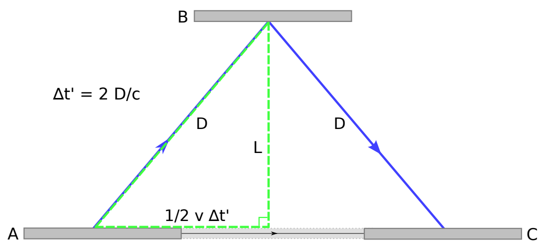
This constancy of the speed of light means, counter to intuition, that speeds of material objects and light are not additive. It is not possible to make the speed of light appear greater by approaching at speed towards the material source that is emitting light. It is not possible to make the speed of light appear less by receding from the source at speed. From one point of view, it is the implications of this unexpected constancy that take away from constancies expected elsewhere.

Consider a simple clock consisting of two mirrors A and B, between which a light pulse is bouncing. The separation of the mirrors is L and the clock ticks once each time the light pulse hits a given mirror.

In the frame where the clock is at rest (diagram at right), the light pulse traces out a path of length $2L$ and the period of the clock is $2L$ divided by the speed of light



Observer at rest measures time 2L/c.



Observer moving parallel relative to setup, measures longer path and thus, with same speed of light c*, measures longer time* 2D/c > 2L/c.
A: location of bottom mirror when signal is generated at time t'=*0.*
B: location of top mirror when signal gets reflected at time t'=D/c.
C: location of bottom mirror when signal hits bottom at time t'=2D/c.

$$\Delta t = \frac{2L}{c}.$$

From the frame of reference of a moving observer traveling at the speed v relative to the rest frame of the clock (diagram at lower right), the light pulse traces out a *longer*, angled path. The second postulate of special relativity states that the speed of light in free space is constant for all inertial observers, which implies a lengthening of the period of this clock from the moving observer's perspective. That is to say, in a frame moving relative to the clock, the clock appears to be running more slowly. Straightforward application of the Pythagorean theorem leads to the well-known prediction of special relativity:

The total time for the light pulse to trace its path is given by

$$\Delta t' = \frac{2D}{c}.$$

The length of the half path can be calculated as a function of known quantities as

$$D = \sqrt{\left(\frac{1}{2}v\Delta t'\right)^2 + L^2}.$$

Substituting D from this equation into the previous and solving for $\Delta t'$ gives:

$$\begin{aligned}
\Delta t' &= \frac{\sqrt{(v\Delta t')^2 + (2L)^2}}{c} \\
(\Delta t')^2 &= \frac{v^2}{c^2}(\Delta t')^2 + \left(\frac{2L}{c}\right)^2 \\
(1 - \frac{v^2}{c^2})(\Delta t')^2 &= \left(\frac{2L}{c}\right)^2 \\
(\Delta t')^2 &= \frac{\left(\frac{2L}{c}\right)^2}{(1 - \frac{v^2}{c^2})} \\
\Delta t' &= \frac{\frac{2L}{c}}{\sqrt{1 - \frac{v^2}{c^2}}}
\end{aligned}$$

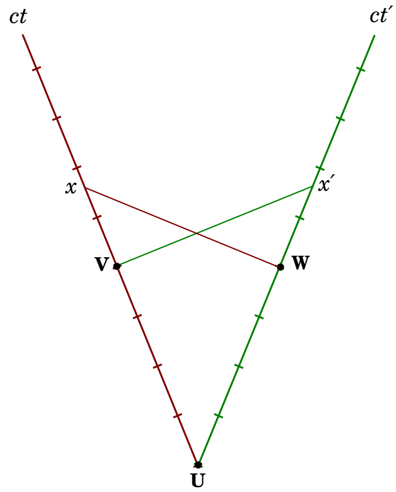
and thus, with the definition of Δt:

$$\Delta t' = \frac{\Delta t}{\sqrt{1 - \frac{v^2}{c^2}}}$$

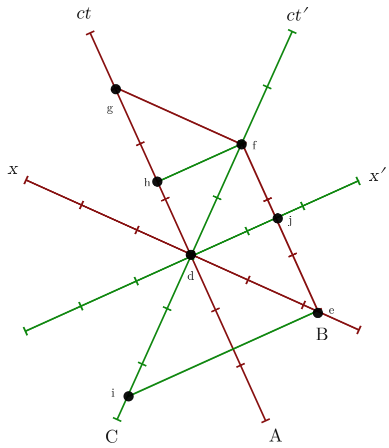
which expresses the fact that for the moving observer the period of the clock is longer than in the frame of the clock itself.

3 Due to relative velocity symmetric between observers

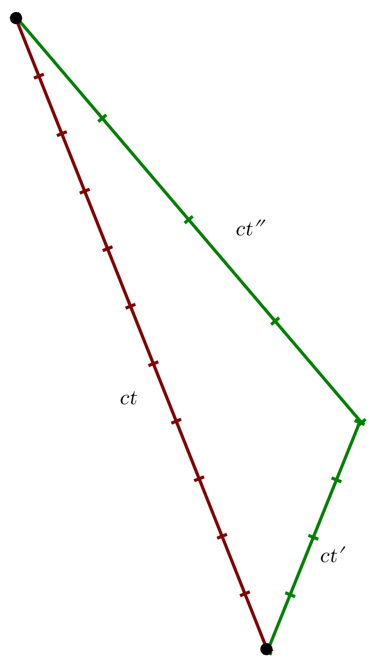
Minkowski diagram



Time UV of a clock in S is shorter compared to Ux′ in S′, and time UW of a clock in S′ is shorter compared to Ux in S.



Clock C in relative motion between two synchronized clocks A and B. C meets A at d, and B at f.



Twin paradox. One twin has to change frames, leading to different proper times in the twin's world lines.

Common sense would dictate that if time passage has slowed for a moving object, the moving object would observe the external world to be correspondingly "sped up". Counterintuitively, special relativity predicts the opposite.

A similar oddity occurs in everyday life. If Sam sees Abigail at a distance she appears small to him and at the same time Sam appears small to Abigail. Being very familiar with the effects of perspective, we see no mystery or a hint of a paradox in this situation.[14]

One is accustomed to the notion of relativity with respect to distance: the distance from Los Angeles to New York is by convention the same as the distance from New York to Los Angeles. On the other hand, when speeds are considered, one thinks of an object as "actually" moving, overlooking that its motion is always relative to something else – to the stars, the ground or to oneself. If one object is moving with respect to another, the latter is moving with respect to the former and with equal relative speed.

In the special theory of relativity, a moving clock is found to be ticking slowly with respect to the observer's clock. If Sam and Abigail are on different trains in near-lightspeed relative motion, Sam measures (by all methods of measurement) clocks on Abigail's train to be running slowly and similarly, Abigail measures clocks on Sam's train to be running slowly.

Note that in all such attempts to establish "synchronization" within the reference system, the question of whether something happening at one location is in fact happening simultaneously with something happening elsewhere, is of key importance. Calculations are ultimately based on determining which events are simultaneous. Furthermore, establishing simultaneity of events separated in space necessarily requires transmission of information between locations, which by itself is an indication that the speed of light will enter the determination of simultaneity.

It is a natural and legitimate question to ask how, in detail, special relativity can be self-consistent if clock C is time-dilated with respect to clock B and clock B is also time-dilated with respect to clock C. It is by challenging the assumptions built into the common notion of simultaneity that logical consistency can be restored. Simultaneity is a relationship between an observer in a particular frame of reference and a set of events. By analogy, left and right are accepted to vary with the position of the observer, because they apply to a relationship. In a similar vein, Plato explained that up and down describe a relationship to the earth and one would not fall off at the antipodes.

In relativity, temporal coordinate systems are set up using a procedure for synchronizing clocks. It is now usually called the *Poincaré-Einstein synchronization procedure*. An observer with a clock sends a light signal out at time t_1 according to his clock. At a distant event, that light signal is reflected back, and arrives back at the observer at time t_2 according to his clock. Since the light travels the same path at the same rate going both out and back for the observer in this scenario, the coordinate time of the event of the light signal being reflected for the observer *t*E is *t*E = (t_1 + t_2) / 2. In this way, a single observer's clock can be used to define temporal coordinates which are good anywhere in the universe.

However, since those clocks are in motion in all other inertial frames, these clock indications are thus not synchronous in those frames, which is the basis of relativity of simultaneity. Because the pairs of putatively simultaneous moments are identified differently by different observers, each can treat the other clock as being the slow one without relativity being self-contradictory. Symmetric time dilation occurs with respect to coordinate systems set up in this manner. It is an effect where another clock is measured to run more slowly than one's own clock. Observers do not consider their own clock time to be affected, but may find that it is observed to be affected in another coordinate system.

3.1 Proper time and Minkowski diagram

This symmetry can be demonstrated in a Minkowski diagram (second image on the right). Clock C resting in inertial frame S′ meets clock A at *d* and clock B at *f* (both resting in S). All three clocks simultaneously start to tick in S. The worldline of A is the ct-axis, the worldline of B intersecting *f* is parallel to the ct-axis, and the worldline of C is the ct′-axis. All events simultaneous with *d* in S are on the x-axis, in S′ on the x′-axis.

The proper time between two events is indicated by a clock present at both events.[15] It is invariant, i.e., in all inertial frames it is agreed that this time is indicated by that clock. Interval *df* is therefore the proper time of clock C, and is shorter with respect to the coordinate times *ef*=*dg* of clocks B and A in S. Conversely, also proper time *ef* of B is shorter with respect to time *if* in S′, because event *e* was measured in S′ already at time *i* due to relativity of simultaneity, long before C started to tick.

From that it can be seen, that the proper time between two events indicated by an unaccelerated clock present at both events, compared with the synchronized coordinate time measured in all other inertial frames, is always the *minimal* time interval between those events. However, the interval between two events can also correspond to the proper time of accelerated clocks present at both events. Under all possible proper times between two events, the proper time of the unaccelerated clock is *maximal*, which is the solution to the twin paradox.[15]

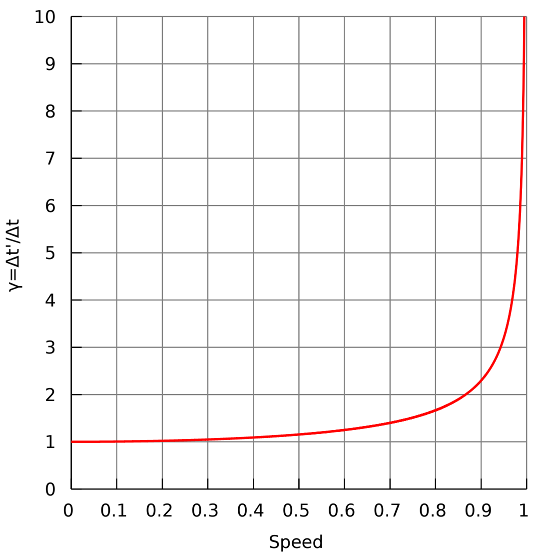
4 Overview of formulae

4.1 Time dilation due to relative velocity

The formula for determining time dilation in special relativity is:

$$\Delta t' = \gamma \, \Delta t = \frac{\Delta t}{\sqrt{1 - \frac{v^2}{c^2}}}$$

where Δ*t* is the time interval between *two co-local events* (i.e. happening at the same place) for an observer in some inertial frame (e.g. ticks on his clock), known as the *proper time*, Δt′ is the time interval between those same



Lorentz factor as a function of speed (in natural units where c = 1). Notice that for small speeds (less than 0.1), γ is approximately 1

events, as measured by another observer, inertially moving with velocity *v* with respect to the former observer, *v* is the relative velocity between the observer and the moving clock, *c* is the speed of light, and the Lorentz factor (conventionally denoted by the Greek letter gamma or γ) is

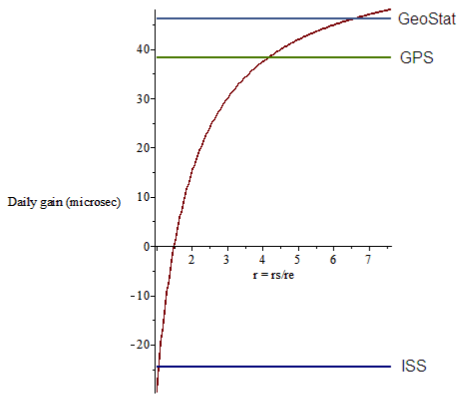
$$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}.$$

Thus the duration of the clock cycle of a moving clock is found to be increased: it is measured to be "running slow". The range of such variances in ordinary life, where $v \ll c$, even considering space travel, are not great enough to produce easily detectable time dilation effects and such vanishingly small effects can be safely ignored for most purposes. It is only when an object approaches speeds on the order of 30,000 km/s (1/10 the speed of light) that time dilation becomes important.

Time dilation by the Lorentz factor was predicted by Joseph Larmor (1897), at least for electrons orbiting a nucleus. Thus "... individual electrons describe corresponding parts of their orbits in times shorter for the [rest] system in the ratio : $\sqrt{1-\frac{v^2}{c^2}}$ " (Larmor 1897). Time dilation of magnitude corresponding to this (Lorentz) factor has been experimentally confirmed, as described below.

4.2 Time dilation due to gravitation and motion together

High accuracy time keeping, low earth orbit satellite tracking, and pulsar timing are applications that require the consideration of the combined effects of mass and motion in producing time dilation. Practical examples include the International Atomic Time standard and its relationship with the Barycentric Coordinate Time standard used for interplanetary objects.



Daily time dilation (gain or loss if negative) in microseconds as a function of (circular) orbit radius r = rs/re*, where* rs *is satellite orbit radius and* re *is the equatorial Earth radius. At* r ≈ *1.497* [Note 1] *there is no time dilation. Here the effects of motion and reduced gravity cancel. ISS astronauts fly below, whereas GPS and Geostationary satellites fly above.*[1]

Relativistic time dilation effects for the solar system and the earth can be modeled very precisely by the Schwarzschild solution to the Einstein field equations. In the Schwarzschild metric, the interval *dt*E is given by[17][18]

$$dt_{\mathrm{E}}^2 = \left(1-\frac{2GM_{\mathrm{i}}}{r_{\mathrm{i}}c^2}\right)dt_{\mathrm{c}}^2 - \left(1-\frac{2GM_{\mathrm{i}}}{r_{\mathrm{i}}c^2}\right)^{-1}\frac{dx^2+dy^2+dz^2}{c^2}$$

where:

*dt*E is a small increment of proper time *t*E (an interval that could be recorded on an atomic clock);

dt_c is a small increment in the coordinate t_c (coordinate time);

dx, *dy* and *dz* are small increments in the three coordinates *x*, *y*, *z* of the clock's position; and

GM_i/r_i represents the sum of the Newtonian gravitational potentials due to the masses in the neighborhood, based on their distances r_i from the clock. This sum GM_i/r_i includes any tidal potentials, and is represented as *U* (using the positive astronomical sign convention for gravitational potentials). The coordinate velocity of the clock is given by

$$v^2 = \frac{dx^2+dy^2+dz^2}{dt_c^2}.$$

The coordinate time t_c is the time that would be read on a hypothetical "coordinate clock" situated infinitely far from all gravitational masses ($U = 0$), and stationary in the system of coordinates ($v = 0$). The exact relation between the rate of proper time and the rate of coordinate time for a clock with a radial component of velocity is

$$\frac{dt_E}{dt_c} = \sqrt{1 - \frac{2U}{c^2} - \frac{v^2}{c^2} - \left(\frac{c^2}{2U} - 1\right)^{-1} \frac{v_{\shortparallel}{}^2}{c^2}}$$

where:

$v\|$ is the radial velocity, and

$U = GM_i/r_i$ is the Newtonian potential, equivalent to half of the escape velocity squared.

The above equation is exact under the assumptions of the Schwarzschild solution.

5 Experimental confirmation

See also: Tests of special relativity

Time dilation has been tested a number of times. The routine work carried on in particle accelerators since the 1950s, such as those at CERN, is a continuously running test of the time dilation of special relativity. The specific experiments include:

5.1 Velocity time dilation tests

- Ives and Stilwell (1938, 1941). The stated purpose of these experiments was to verify the time dilation effect, predicted by Larmor–Lorentz ether theory, due to motion through the ether using Einstein's suggestion that Doppler effect in canal rays would provide a suitable experiment. These experiments measured the Doppler shift of the radiation emitted from cathode rays, when viewed from directly in front and from directly behind. The high and low frequencies detected were not the classically predicted values

  $$\frac{f_0}{1 - v/c} \quad \text{and} \quad \frac{f_0}{1 + v/c}.$$

  The high and low frequencies of the radiation from the moving sources were measured as[19]

  $$\sqrt{\frac{1 + v/c}{1 - v/c}} f_0 = \gamma (1 + v/c) f_0 \quad \text{and} \quad \sqrt{\frac{1 - v/c}{1 + v/c}} f_0 = \gamma (1 - v/c) f_0.$$

  as deduced by Einstein (1905) from the Lorentz transformation, when the source is running slow by the Lorentz factor.

- Rossi and Hall (1941) compared the population of cosmic-ray-produced muons at the top of a mountain to that observed at sea level. Although the travel time for the muons from the top of the mountain to the base is several muon half-lives, the muon sample at the base was only moderately reduced. This is explained by the time dilation attributed to their high speed relative to the experimenters. That is to say, the muons were decaying about 10 times slower than if they were at rest with respect to the experimenters.

- Hasselkamp, Mondry, and Scharmann[20] (1979) measured the Doppler shift from a source moving at right angles to the line of sight. The most general relationship between frequencies of the radiation from the moving sources is given by:

  $$f_{\text{detected}} = f_{\text{rest}} \left(1 - \frac{v}{c} \cos\phi\right) / \sqrt{1 - v^2/c^2}$$

  as deduced by Einstein (1905).[21] For $\phi = 90°$ ($\cos\phi = 0$) this reduces to $f_{\text{detected}} = f_{\text{rest}}\gamma$. This lower frequency from the moving source can be attributed to the time dilation effect and is often called the transverse Doppler effect and was predicted by relativity.

- In 2010 time dilation was observed at speeds of less than 10 meters per second using optical atomic clocks connected by 75 meters of optical fiber.[22]

5.2 Gravitational time dilation tests

- In 1959 Robert Pound and Glen A. Rebka measured the very slight gravitational red shift in the frequency of light emitted at a lower height, where Earth's gravitational field is relatively more intense. The results were within 10% of the predictions of general relativity. In 1964, Pound and J. L. Snider measured a result within 1% of the value predicted by gravitational time dilation.[23] (See Pound–Rebka experiment)

- In 2010 gravitational time dilation was measured at the earth's surface with a height difference of only one meter, using optical atomic clocks.[22]

5.3 Velocity and gravitational time dilation combined-effect tests

- Hafele and Keating, in 1971, flew caesium atomic clocks east and west around the earth in commercial airliners, to compare the elapsed time against that of a clock that remained at the U.S. Naval Observatory. Two opposite effects came into play. The clocks were expected to age more quickly (show a larger elapsed time) than the reference clock, since they were in a higher (weaker) gravitational potential for most of the trip (c.f. Pound–Rebka experiment). But also, contrastingly, the moving clocks were expected to age more slowly because of the speed of their travel. From the actual flight paths of each trip, the theory predicted that the flying clocks, compared with reference clocks at the U.S. Naval Observatory, should have lost 40±23 nanoseconds during the eastward trip and should have gained 275±21 nanoseconds during the westward trip. Relative to the atomic time scale of the U.S. Naval Observatory, the flying clocks lost 59±10 nanoseconds during the eastward trip and gained 273±7 nanoseconds during the westward trip (where the error bars represent standard deviation).[24] In 2005, the National Physical Laboratory in the United Kingdom reported their limited replication of this experiment.[25] The NPL experiment differed from the original in that the caesium clocks were sent on a shorter trip (London–Washington, D.C. return), but the clocks were more accurate. The reported results are within 4% of the predictions of relativity, within the uncertainty of the measurements.

- The Global Positioning System can be considered a continuously operating experiment in both special and general relativity. The in-orbit clocks are corrected for both special and general relativistic time dilation effects as described above, so that (as observed from the earth's surface) they run at the same rate as clocks on the surface of the Earth.[26]

5.4 Muon lifetime

A comparison of muon lifetimes at different speeds is possible. In the laboratory, slow muons are produced, and in the atmosphere very fast moving muons are introduced by cosmic rays. Taking the muon lifetime at rest as the laboratory value of 2.197 μs, the lifetime of a cosmic ray produced muon traveling at 98% of the speed of light is about five times longer, in agreement with observations.[27] In the muon storage ring at CERN the lifetime of muons circulating with $\gamma = 29.327$ was found to be dilated to 64.378 μs, confirming time dilation to an accuracy of 0.9 ± 0.4 parts per thousand.[28] In this experiment the "clock" is the time taken by processes leading to muon decay, and these processes take place in the moving muon at its own "clock rate", which is much slower than the laboratory clock.

6 Space flight

Time dilation would make it possible for passengers in a fast-moving vehicle to travel further into the future while aging very little, in that their great speed slows down the passage of on-board time relative to that of an observer. That is, the ship's clock (and according to relativity, any human traveling with it) shows less elapsed time than the clocks of observers on earth. For sufficiently high speeds the effect is dramatic.[2] For example, one year of travel might correspond to ten years at home. Indeed, a constant 1 g acceleration would permit humans to travel through the entire known Universe in one human lifetime.[29] The space travelers could return to Earth billions of years in the future. A scenario based on this idea was presented in the novel *Planet of the Apes* by Pierre Boulle.

A more likely use of this effect would be to enable humans to travel to nearby stars without spending their entire lives aboard a ship. However, any such application of time dilation during interstellar travel would require the use of some new, advanced method of propulsion. The Orion Project has been the only major attempt toward this idea.

Current space flight technology has fundamental theoretical limits based on the practical problem that an increasing amount of energy is required for propulsion as a craft approaches the speed of light. The likelihood of collision with small space debris and other particulate material is another practical limitation. At the velocities presently attained, however, time dilation occurs but is too small to be a factor in space travel. Travel to regions of spacetime where gravitational time dilation is taking place, such as within the gravitational field of a black hole but outside the event horizon (perhaps on a hyperbolic trajectory exiting the field), could also yield results consistent with present theory.

6.1 Time dilation at constant force

In special relativity, time dilation is most simply described in circumstances where relative velocity is unchanging. Nevertheless, the Lorentz equations allow one to calculate proper time and movement in space for the simple case of a spaceship which is applied with a force per unit mass, relative to some reference object in uniform (i.e. constant velocity) motion, equal to g throughout the period of measurement.

Let t be the time in an inertial frame subsequently called the rest frame. Let x be a spatial coordinate, and let the direction of the constant acceleration as well as the spaceship's velocity (relative to the rest frame) be parallel to the x-axis. Assuming the spaceship's position at time $t = 0$ being $x = 0$ and the velocity being v_0 and defining the following abbreviation

$$\gamma_0 = \frac{1}{\sqrt{1 - v_0^2/c^2}},$$

the following formulas hold:[30]

Position:

$$x(t) = \frac{c^2}{g}\left(\sqrt{1+\frac{(gt+v_0\gamma_0)^2}{c^2}}-\gamma_0\right).$$

Velocity:

$$v(t) = \frac{gt+v_0\gamma_0}{\sqrt{1+\frac{(gt+v_0\gamma_0)^2}{c^2}}}.$$

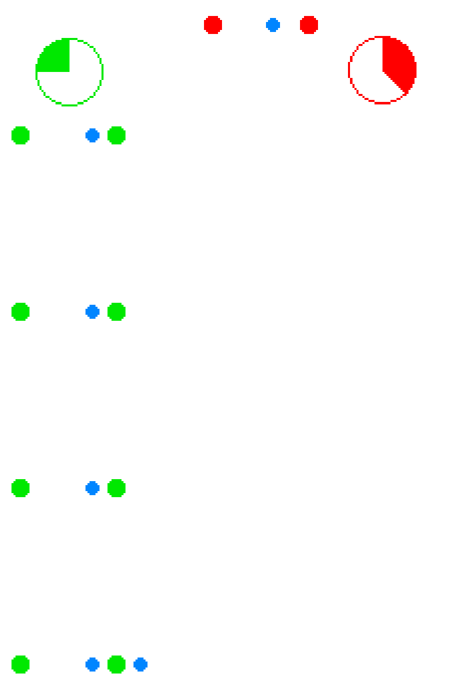
Proper time:

$$\tau(t) = \tau_0 + \int_0^t \sqrt{1-\left(\frac{v(t')}{c}\right)^2}\,dt'.$$

In the case where $v(0) = v_0 = 0$ and $\tau(0) = \tau_0 = 0$ the integral can be expressed as a logarithmic function or, equivalently, as an inverse hyperbolic function:

$$\tau(t) = \frac{c}{g}\ln\left(\frac{gt}{c}+\sqrt{1+\left(\frac{gt}{c}\right)^2}\right) = \frac{c}{g}\operatorname{arsinh}\left(\frac{gt}{c}\right).$$

6.2 Spacetime geometry of velocity time dilation



Time dilation in transverse motion

The green dots and red dots in the animation represent spaceships. The ships of the green fleet have no velocity relative to each other, so for the clocks on board the individual ships the same amount of time elapses relative to each other, and they can set up a procedure to maintain a synchronized standard fleet time. The ships of the "red fleet" are moving with a velocity of 0.866c with respect to the green fleet.

The blue dots represent pulses of light. One cycle of light-pulses between two green ships takes two seconds of "green time", one second for each leg.

As seen from the perspective of the reds, the transit time of the light pulses they exchange among each other is one second of "red time" for each leg. As seen from the perspective of the greens, the red ships' cycle of exchanging light pulses travels a diagonal path that is two light-seconds long. (As seen from the green perspective the reds travel 1.73 ($\sqrt{3}$) light-seconds of distance for every two seconds of green time.)

One of the red ships emits a light pulse towards the greens every second of red time. These pulses are received by ships of the green fleet with two-second intervals as measured in green time. Not shown in the animation is that all aspects of physics are proportionally involved. The light pulses that are emitted by the reds at a particular frequency as measured in red time are received at a lower frequency as measured by the detectors of the green fleet that measure against green time, and vice versa.

The animation cycles between the green perspective and the red perspective, to emphasize the symmetry. As there is no such thing as absolute motion in relativity (as is also the case for Newtonian mechanics), both the green and the red fleet are entitled to consider themselves motionless *in their own frame of reference*.

Again, it is vital to understand that the results of these interactions and calculations reflect the real state of the ships as it emerges from their situation of relative motion. It is not a mere quirk of the method of measurement or communication.

7 See also

- Mass dilation
- Length contraction
- Albert Einstein

8 Footnotes

[1] Average time dilation has a weak dependence on the orbital inclination angle (Ashby 2003, p.32). The $r \approx 1.497$ result corresponds to [16] the orbital inclination of modern GPS satellites, which is 55 degrees.

9 References

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[5] For sources on special relativistic time dilation, see Albert Einstein's own popular exposition, published in English translation (1920) as Einstein, Albert (1920). "On the Idea of Time in Physics". *Relativity: The Special and General Theory*. Henri Holt. ISBN 1-58734-092-5. and also in sections 9–12. See also the articles Special relativity, Lorentz transformation and Relativity of simultaneity.

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11 External links

- Online Time Dilation Calculator
- Proper Time
- Merrifield, Michael. "Lorentz Factor (and time dilation)". *Sixty Symbols*. Brady Haran for the University of Nottingham.

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